# THE PROBLEM OF METHODS OF GENERALIZING EXPERIMENTAL DATA ON HEAT TRANSFER AND RESISTANCE IN GRADIENT FLOWS

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Generalized equations are proposed for the heat transfer and resistance in a turbulent flow of a gas along a flat asymmetrical channel of the diffusor–confusor type with rounded edges.

The results of an experimental investigation of heat transfer and resistance for flat asymmetric channels of diffusor–confusor type were given in [1, 2].

The problem arises of the representation of the experimental results on heat transfer and resistance for the gradient flows of very complex type considered there. Similarity theory is fundamental to the solution of this problem.

It is quite obvious that in this problem the hydrodynamical conditions, and, consequently, the intensity of heat transfer, are determined by the flow-condition criterion, Re, and by the shape of the channel, which is characterized by the following set of geometrical criteria of parametric type:

$$\frac{a_c}{a_0},\ \frac{b}{a_0},\ \frac{c}{a_0},\ \frac{h}{a_0}.$$

The parametric criterion $h/a_0$ can be dropped, since in our experiments $d_e = (1.72-1.89)a_0$, i.e. $d_e \approx 2a_0$, and it is sufficiently accurate to assume that $h \gg a_0$. When the gas moves in conditions in which heat transfer occurs, a temperature factor has to be introduced into the number of arguments. But in our case the temperature factor is virtually constant and we take no account of it.

Thus, for our problem, the generalized equations for heat transfer and resistance have the form:

$$\mathrm{Nu} = \mathrm{Nu}\left(\mathrm{Re},\ \frac{a_c}{a_0},\ \frac{b}{a_0},\ \frac{c}{a_0}\right), \tag{1}$$

$$\zeta = \zeta\left(\mathrm{Re},\ \frac{a_c}{a_0},\ \frac{b}{a_0},\ \frac{c}{a_0}\right) \tag{2}$$

(the Prandtl number Pr does not occur since we are only studying gas flow). The effect of the channel geometry on the intensity of heat transfer and resistance appears directly in terms of the pressure gradients and the duration of their action. Hence it is expedient to transform to arguments indirectly defining their effects.

The effect of pressure gradients of different signs can be taken into account by nondimensional pressure gradients averaged with respect to the diffusor and confusor lengths:

$$\Gamma_1^+ \equiv \overline{\frac{d\left(\frac{p}{\rho U_a^2}\right)^+}{d\left(\frac{x}{a_0}\right)}} \text{ and } \Gamma_1^- \equiv \overline{\frac{d\left(\frac{p}{\rho U_a^2}\right)^-}{d\left(\frac{x}{a_0}\right)}}.$$

For an ideal (frictionless) flow along a nonsymmetric channel of diffusor–confusor type we have

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TABLE 1. Geometrical Characteristics of the Channels Investigated (Diffusor – confusor type with rounded edges, $a_0$ = 16.8 mm, 33.3 mm, 47.7 mm)

| γ, deg | φ, deg | b, mm | c, mm | b:c | γ, deg | φ, deg | b, mm | c, mm | b:c |
|---|---|---|---|---|---|---|---|---|---|
| 12 | 47 | 40 | 8 | 5 : 1 | 8 | 35 | 40 | 8 | 5 : 1 |
| 12 | 23 | 40 | 20 | 2 : 1 | 8 | 16 | 40 | 20 | 2 : 1 |
| 12 | 12 | 40 | 40 | 1 : 1 | 8 | 8 | 40 | 40 | 1 : 1 |
| 12 | 6 | 40 | 80 | 1 : 2 | 6 | 12 | 80 | 40 | 2 : 1 |
| 12 | 4 | 40 | 120 | 1 : 3 | 4 | 12 | 120 | 40 | 3 : 1 |

$$\Gamma_1^+ \equiv -\frac{a_a^3 \operatorname{tg}\gamma}{a_0 \cdot a_c^2} \text{ and } \Gamma_1^- \equiv \frac{a_a^3 \operatorname{tg}\varphi}{a_0 a_c^2},$$

where

$$a_a = a_0 + \frac{b}{2}\operatorname{tg}\gamma = a_0 + \frac{c}{2}\operatorname{tg}\varphi,\quad a_c = a_0 + b\operatorname{tg}\gamma = a_0 + c\operatorname{tg}\varphi.$$

The duration of the pressure gradients can be taken into account by introducing a parametric criterion, for example, $b/a_0$, since for investigations of channels we have

$$\frac{c}{b} = \frac{\Gamma_1^+}{\Gamma_1^-}.$$

Then the generalized equations (1) and (2) take the forms

$$\mathrm{Nu} = \mathrm{Nu}\left(\mathrm{Re},\ \Gamma_1^+,\ \Gamma_1^-,\ \frac{b}{a_0}\right), \tag{3}$$

$$\zeta = \zeta\left(\mathrm{Re},\ \Gamma_1^+,\ \Gamma_1^-,\ \frac{b}{a_0}\right). \tag{4}$$

If we make a transition from the natural scale of length $a_0$ to the physical $\nu/U_a$, we obtain modified expressions for the nondimensional pressure gradients, averaged with respect to the diffusor – confusor length:

$$\Gamma^+ \equiv \overline{\frac{d\left(\frac{p}{\rho U_a^2}\right)^+}{d\left(\frac{x}{\nu/U_a}\right)}} = \frac{\nu}{\rho U_a^3}\cdot\overline{\frac{dp^+}{dx}} \text{ and } \Gamma^- \equiv \overline{\frac{d\left(\frac{p}{\rho U_a^2}\right)^-}{d\left(\frac{x}{\nu/U_a}\right)}} = \frac{\nu}{\rho U_a^3}\cdot\overline{\frac{dp^-}{dx}}.$$

These are essentially identical to the parameter

$$K \equiv \frac{d\left(\frac{p}{\rho U_\infty^2}\right)}{d\left(\frac{x}{\nu/U_\infty}\right)} = -\frac{\nu}{\rho U_\infty^3}\cdot\frac{dp}{dx},$$

which has been used in recent papers [3-5] as a characteristic of the processes of generation and degeneration of turbulence in a gradient flow past a flat plate. For an ideal (frictionless) flow in an asymmetrical channel of diffusor – confusor type we obtain

$$\Gamma^+ \equiv -\frac{d_e \operatorname{tg}\gamma}{d_0\,\mathrm{Re}} \text{ and } \Gamma^- \equiv \frac{d_e \operatorname{tg}\varphi}{a_0 \mathrm{Re}}.$$

Thus, the generalized equations take the forms:

$$\mathrm{Nu} = \mathrm{Nu}\left(\mathrm{Re},\ \Gamma^+,\ \Gamma^-,\ \frac{b}{a_0}\right), \tag{5}$$

$$\zeta = \zeta\left(\mathrm{Re},\ \Gamma^+,\ \Gamma^-,\ \frac{b}{a_0}\right). \tag{6}$$

Experimental results on heat transfer and resistance in the channels investigated (see Table 1) agree to a sufficiently high degree of accuracy with the following criterial equations:

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_0} = 1.2\left(\frac{\Gamma^-}{\Gamma^{-6}}\right)^n\left(\frac{b}{a_0}\right)^{-0.200}, \tag{7}$$

$$\frac{\zeta}{\zeta_0} = 0.8\left(\frac{\Gamma^-}{10^{-6}}\right)^m\left(\frac{b}{a_0}\right)^{-0.175}, \tag{8}$$

where

$$\mathrm{Nu}_0 = 0.018\mathrm{Re}^{0.8};\ n = (0.012 + 0.00104\mathrm{Re}\cdot 10^{-4})\,\gamma;$$

$$\zeta_0 = 0.3164\mathrm{Re}^{-0.25};\ m = (0.022 + 0.00140\mathrm{Re}\cdot 10^{-4})\,\gamma.$$

The mean gas temperature was taken as the definitive temperature and $d_e$ as the typical dimension.

In (7) and (8) the experimental results on the intensity of heat transfer and resistance are given as proportions of the corresponding computed values for flow in a channel of constant cross section along its length at a fixed value of Re, which makes it possible explicitly to estimate the effect of the factors which are characteristic of flows in a longitudinal pressure field of variable sign.

In the experiments $\Gamma^+$ varied in the interval $(-1.51\text{–}4.04)\cdot 10^{-6}$, $\Gamma^-$ in the interval $(1.51\text{–}176)\cdot 10^{-6}$. The effect of $\Gamma^-$ was thus clearly distinguished, while the effect of $\Gamma^+$ appeared in implicit form, i.e., in terms of the dependence of the power indices n and m on $\gamma$ and Re.

For convenience of expression the parameter $\Gamma^-$ is referred to the quantity $10^{-6}$ (i.e., the ratio $\Gamma^-/10^{-6}$ is quoted), this being of the same order of magnitude as $K_{cr} = 2\cdot 10^{-6}$, which corresponds to the effect of the laminarization of the turbulent boundary layer [5].

Equations (7) and (8) are valid for Re = $(10\text{–}80)\cdot 10^3$; $b/a = 0.87\text{–}7.15$; $\gamma = 4\text{–}12°$. In addition, (7) is applicable for $\Gamma^-/10^{-6} > 1.51$ and $\mathrm{Re}/10^4(\Gamma^-/10^{-6}) < 10\gamma$, while (8) is applicable for $1.5k(\Gamma^-/10^{-6}) < 176$. This difference in the boundaries within which (7) and (8) are applicable is because for the channels investigated an increase in the diffusor–confusor ratio, with $\gamma$ fixed (i.e., an increase in $\Gamma^-$) leads to a continuous increase in $\zeta$. Then the intensity of heat transfer increases only up to some limit, after which it begins to decrease. This is confirmation that when turbulence generated in the diffusor is "operative," the confusor must have some particular length.

## NOTATION

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|---|---|
| Re | is the Reynolds number; |
| Pr | is the Prandtl number; |
| Nu | is the Nusselt number; |
| $\zeta$ | is the hydrodynamic resistance coefficient; |
| $a_0$, $a_a$, $a_c$ | are the width of diffusor entrance, mean diffusor width, and width of diffusor exit respectively; |
| b | is the diffusor length; |
| c | is the confusor length; |
| x | is the coordinate in flow direction; |
| h | is the channel height; |
| $d_e$ | is the equivalent diameter of diffusor entrance section; |
| $\gamma$ | is the diffusor angle of expansion; |
| $\varphi$ | is the confusor contraction angle; |
| $\rho$ | is the gas density; |
| $\nu$ | is the kinematic viscosity coefficient; |
| p | is the static pressure; |
| $U_a$ | is the gas velocity averaged with respect to flow rate; |
| $U_\infty$ | is the potential flow velocity; |
| $\Gamma_1^+$, $\Gamma_1^-$ | are the modified expressions for the nondimensional pressure gradients averaged with respect to diffusor and confusor lengths; |
| K, $K_{cr}$ | are the pressure gradient parameter and its critical value. |

## LITERATURE CITED


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